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Eigenvalue approach in a three-dimensional generalized thermoelastic interactions with temperature-dependent material properties

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ABSTRACT

A three-dimensional model of the generalized thermoelasticity without energy dissipation under temperature-dependent mechanical properties is established. The modulus of elasticity is taken as a linear function of the reference temperature. The resulting formulation in the context of Green and Naghdi model II is applied to a half-space subjected to a time-dependent heat source and traction free surface. The normal mode analysis and eigenvalue approach techniques are used to solve the resulting non-dimensional coupled equations. Numerical results for the field quantities are given in the physical domain and illustrated graphically. The results are also compared to results obtained in the case of temperature-independent modulus of elasticity.

1. Introduction

In fact, the change of body temperature has an effect on the strain/stress fields and conversely, i.e. mechanical action, and corresponding strain produce a temperature field. The numerical value of thermal conductivity varies with temperature, especially if a region of change of temperature is large. So, the thermal conductivity, and may be the heat capacity, should be considered temperature-dependent in most of the practical engineering problems. Linear elasticity is at the heart of almost all continuum-based constitutive models used in structural and geotechnical engineering, and therefore it is reasonable to concentrate efforts (initially at least) on improvement of solvers for these cases. While in service, structural elements are frequently subjected to not only force loads but also non-uniform heating causing thermal stresses. These stress themselves or in combination with mechanical stresses due to external loads may cause the material to fracture. Therefore, to perform a complete strength analysis of structures, it is necessary to know the magnitude and distribution of thermal stresses. In this connection, issues associated with the determination of temperature fields and thermal stresses are of importance and draw the attention of experts of different professions. The theory of couple thermoelasticity was extended by Lord and Shulman (LS) [1] and Green and Lindsay (GL) [2] by including the thermal relaxation time in constitutive relations. The anisotropic case was later developed by Dhaliwal and Sherief [3]. In the work reported by Green and Naghdi (GN), they formulated three models of thermoelasticity for the homogeneous and isotropic materials [4,5], which are labeled as models I, II, and III. GNII model is due to Green and Naghdi [5], whose significance is that the internal rate of production of entropy is taken to be identically zero, i.e., there is no dissipation of thermal energy. This theory is known as the theory of thermoelasticity without

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energy dissipation. The thermoelasticity theories cited above (LS, GL, and GN theories) are also known as the generalized thermoelasticity theories or thermoelasticity theories with finite thermal wave speed.

In most of the problems, the material properties of the medium are taken to be constant. Modern structure elements are often subjected to temperature changes of such magnitude that their material properties may no longer be regarded as having constant values even in approximate sense. The thermal and mechanical properties of materials vary with temperature, so that the temperature dependence of material properties must be taken into consideration in the thermal stress analysis of these elements. Suhara [6] solved the thermoelastic problems of hollow circular cylinder of which only the shear modulus was temperature-dependent. Since his study, many investigators studied temperature-dependence. Youssef [7–9] has many contributions for temperature-dependent properties of materials. Ezzat et al. [10] investigated problem in generalized thermoelasticity with the modulus of elasticity dependent with temperature. Abbas and Othman [11] studied thermal shock problem in an isotropic hollow cylinder and temperature dependent elastic moduli. Subsequently, several investigations [12–17] are carried out based on different generalized theories of thermoelasticity in one and two-dimensional problems. Many problems in engineering practice involve determination of stresses and/or displacements in bodies that are three-dimensional. Exact analytical solutions are available only for a few three-dimensional problems [18–21] with simple geometries and/or loading conditions. Hence, numerical or experimental analysis is generally required in solving such problems.

In spite of these works, relatively less attention has been given to three-dimensional problems of thermoelasticity. The main objective of this present study is to investigate the basis form of the particular solution to the three-dimensional thermoelasticity problem when the modulus of elasticity is taken as a linear function of reference temperature in the context of Green and Naghdi model II in absence of body force and heat sources. The governing coupled equations in Cartesian coordinates are applied to time dependent heat source on the free surface. The normal mode analysis and eigenvalue approach techniques are used to solve the resulting non-dimensional coupled equations. Numerical results for temperature, displacement distributions and thermal stresses, are presented graphically and discussed.

2. Governing equations

We consider an isotropic, homogeneous, linear and thermally elastic medium with temperature dependent mechanical properties. The basic equations in the context of the Green and Naghdi model II in absence of body forces and heat source are given by

1. The strain–displacement relations

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{1}$$

2. The constitutive law for the theory of generalized thermoelasticity

$$\sigma_{ij} = 2\mu e_{ij} + \left[\lambda e - \gamma \left(T - T_o\right)\right]\delta_{ij}. \tag{2}$$

3. The equation of motion

$$\sigma_{ij,j} = \rho \ddot{u}_i. \tag{3}$$

4. The generalized heat conduction equation in the Green and Naghdi model II

$$\left(KT_{,i}\right)_{,i} = \frac{\partial^2}{\partial t^2}\left(\rho c_e T + \gamma T_o e\right), \tag{4}$$

where $i, j = 1, 2, 3$ refer to general coordinates; ρ is the mass density; T the temperature change of a material particle; T_o the reference uniform temperature of the body; u_i the displacement vector components; e_{ij} the strain tensor; σ_{ij} the stress tensor; c_E the specific heat at constant strain; γ the thermal elastic coupling tensor in which $\gamma = (3\lambda + 2\mu)\,\alpha_t$, α_t is the coefficient of linear thermal expansion; K is a material constant characteristic of the theory; λ, μ are elastic parameters. For temperature dependent material, we will suppose that

$$\lambda = \lambda_o f(T), \qquad \mu = \mu_o f(T), \qquad K = K_o f(T), \qquad \gamma = \gamma_o f(T), \tag{5}$$

where λ_o, μ_o, k_o, and γ_o are considered constants, $f(T)$ is given a non-dimensional function of temperature. In the case of temperature-independent material properties $f(T) = 1$ and $\lambda = \lambda_o$, $\mu = \mu_o$, $K = K_o$, and $\gamma = \gamma_o$.

3. Problem formulation

Consider an isotropic, homogeneous and elastic body in three-dimensional fills the region $\psi = \{x, y, z : 0 \le x < \infty, -\infty < y < \infty, -\infty < z < \infty\}$, subjected to a time-dependent heat source and traction free on the bounding plane to the surface $x = 0$. The governing equations will be written in the context of Green and Naghdi model II with temperature dependent mechanical properties when the body has no heat sources or any body forces, and we will use the Cartesian co-ordinates (x, y, z) and the components of the displacement $u_i = (u, v, w)$ to write them as follows:

The strain–displacement relations are in the forms

$$e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial v}{\partial y}, \qquad e_{zz} = \frac{\partial w}{\partial z}, \qquad e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$
$$e_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \qquad e_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right). \tag{6}$$

The constitutive relations are in the forms

$$\sigma_{xx} = [2\mu e_{xx} + \lambda e - \gamma\,(T - T_o)]f(T),$$
$$\sigma_{yy} = \left[2\mu e_{yy} + \lambda e - \gamma\,(T - T_o)\right]f(T),$$
$$\sigma_{zz} = [2\mu e_{zz} + \lambda e - \gamma\,(T - T_o)]f(T),$$
$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)f(T),$$
$$\sigma_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)f(T),$$
$$\sigma_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)f(T). \tag{7}$$

The equations of motion

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{9}$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2}. \tag{10}$$

The heat conduction equation

$$K_o\left[\frac{\partial}{\partial x}\left(f(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(f(T)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(f(T)\frac{\partial T}{\partial z}\right)\right] = \frac{\partial^2}{\partial t^2}\left(\rho c_e T + \gamma_o f T_o e\right), \tag{11}$$

where $e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

Rishin et al. [22] investigated the relationship between modulus of elasticity of several sprayed coatings and temperature, and they found the modulus of elasticity decreases monotonically with the increasing of temperature. For simplicity and without loss of generality, we assume:

$$f(T) = 1 - \alpha^* T, \tag{12}$$

where $\alpha^*\left[k^{-1}\right]$ is an empirical material constant. In generalized thermoelasticity, as well as in the coupled theory only the infinitesimal temperature deviations from reference temperature are considered. For linearity of the governing partial differential equations of the problem, we have to take into account the condition $\frac{|T-T_o|}{T_o} \ll 1$, which gives the approximating function of $f(T)$ to be in the form:

$$f(T) = 1 - \alpha^* T_o. \tag{13}$$

For our convenience, the following non-dimensional variables and notations are used:

$$(x', y', z') = \frac{1}{l}(x, y, z), \qquad \left(u', v', w'\right) = \frac{\lambda_o + 2\mu_o}{\gamma_o T_o l}\,(u, v, w), \qquad t' = \frac{ct}{l},$$
$$T' = \frac{T - T_o}{T_o}, \qquad \sigma'_{ij} = \frac{\sigma_{ij}}{\gamma_o T_o}, \tag{14}$$

where l standard length and $c^2 = \frac{\lambda_o + 2\mu_o}{\rho}$. In terms of the non-dimensional quantities defined in Eq. (14), the above governing equations reduce to (dropping the dashed for convenience)

$$\frac{\partial^2 u}{\partial x^2} + \xi\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + (1 - \xi)\,\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{\partial T}{\partial x} = \alpha\frac{\partial^2 u}{\partial t^2}, \tag{15}$$